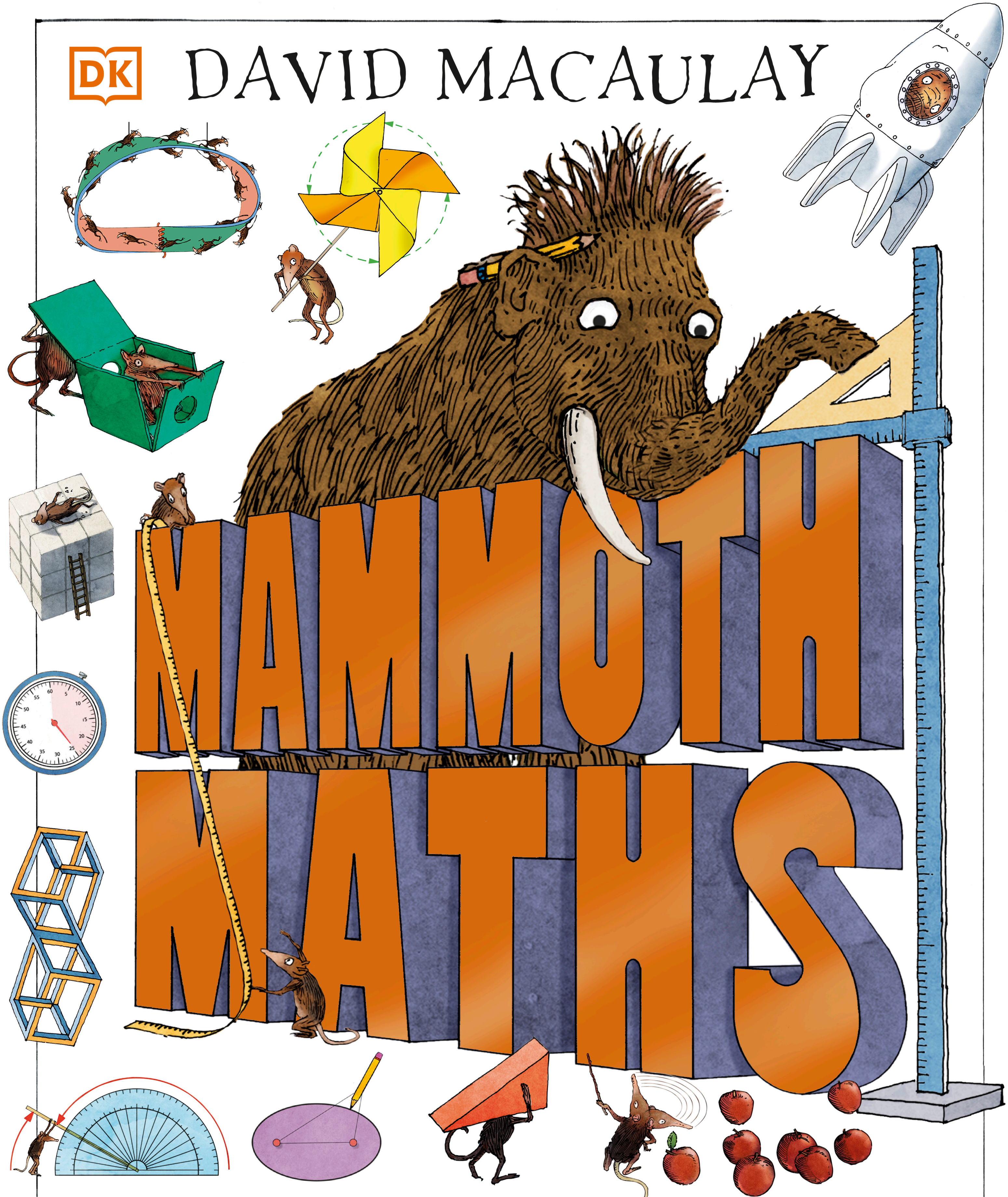
DK

DAVID MACAULAY

MAMMOTH MATHS



EVERYTHING YOU NEED TO KNOW ABOUT NUMBERS

DEMONSTRATED BY MAMMOTHS

Mammoth Maths

About this eBook

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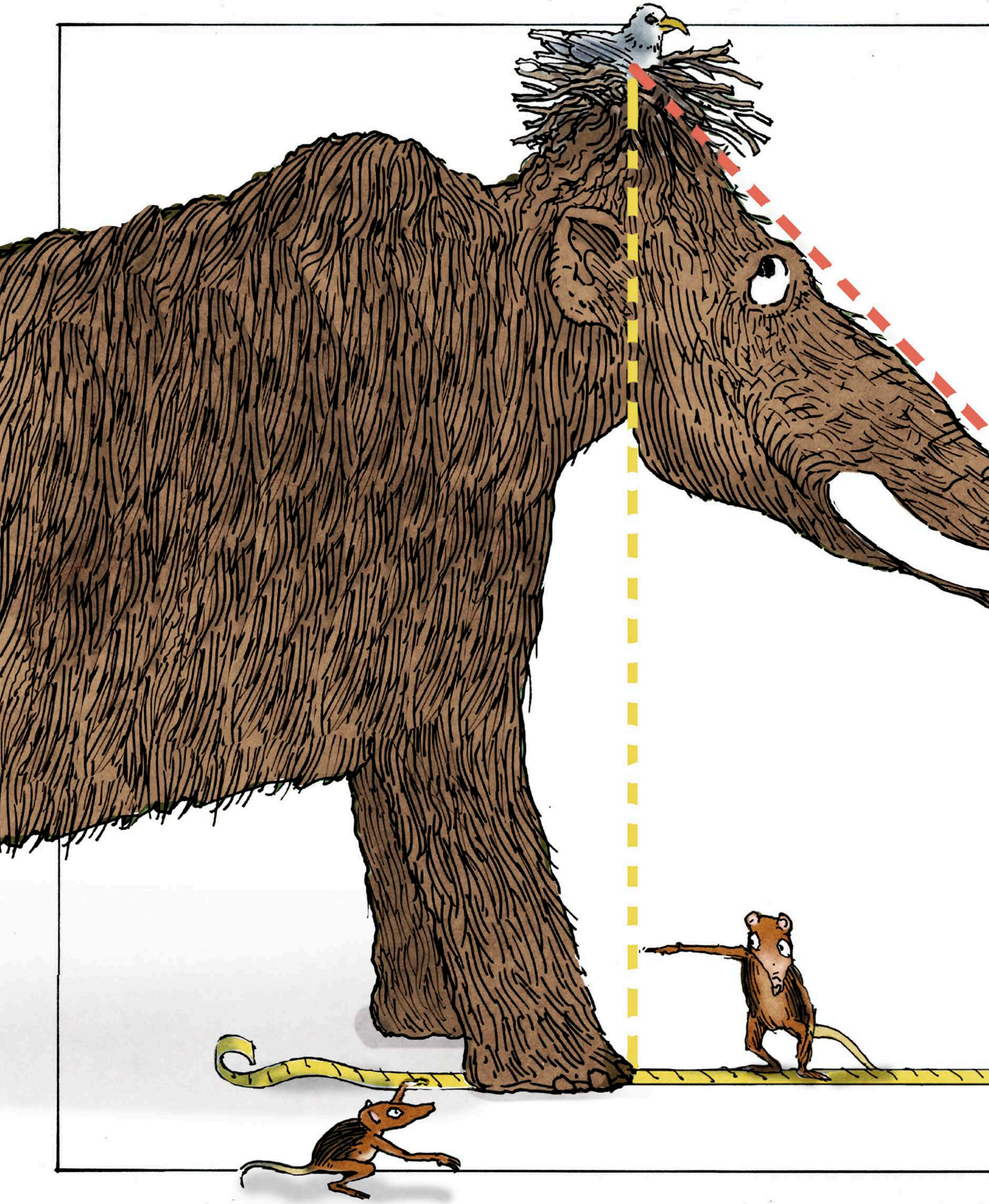
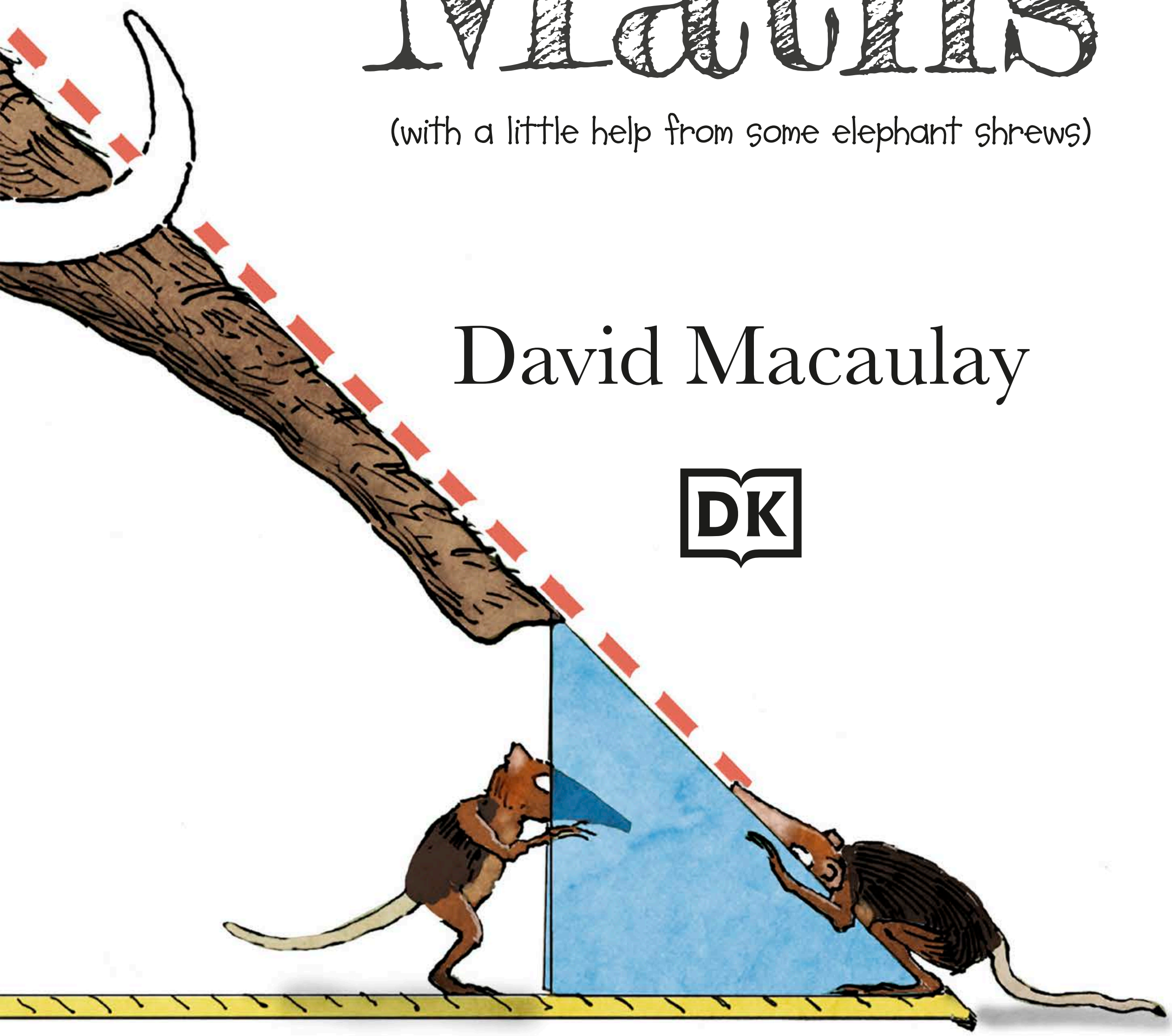


Mammoth Maths

(with a little help from some elephant shrews)



David Macaulay

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Text by Rona Skene
Consultant Branka Surla

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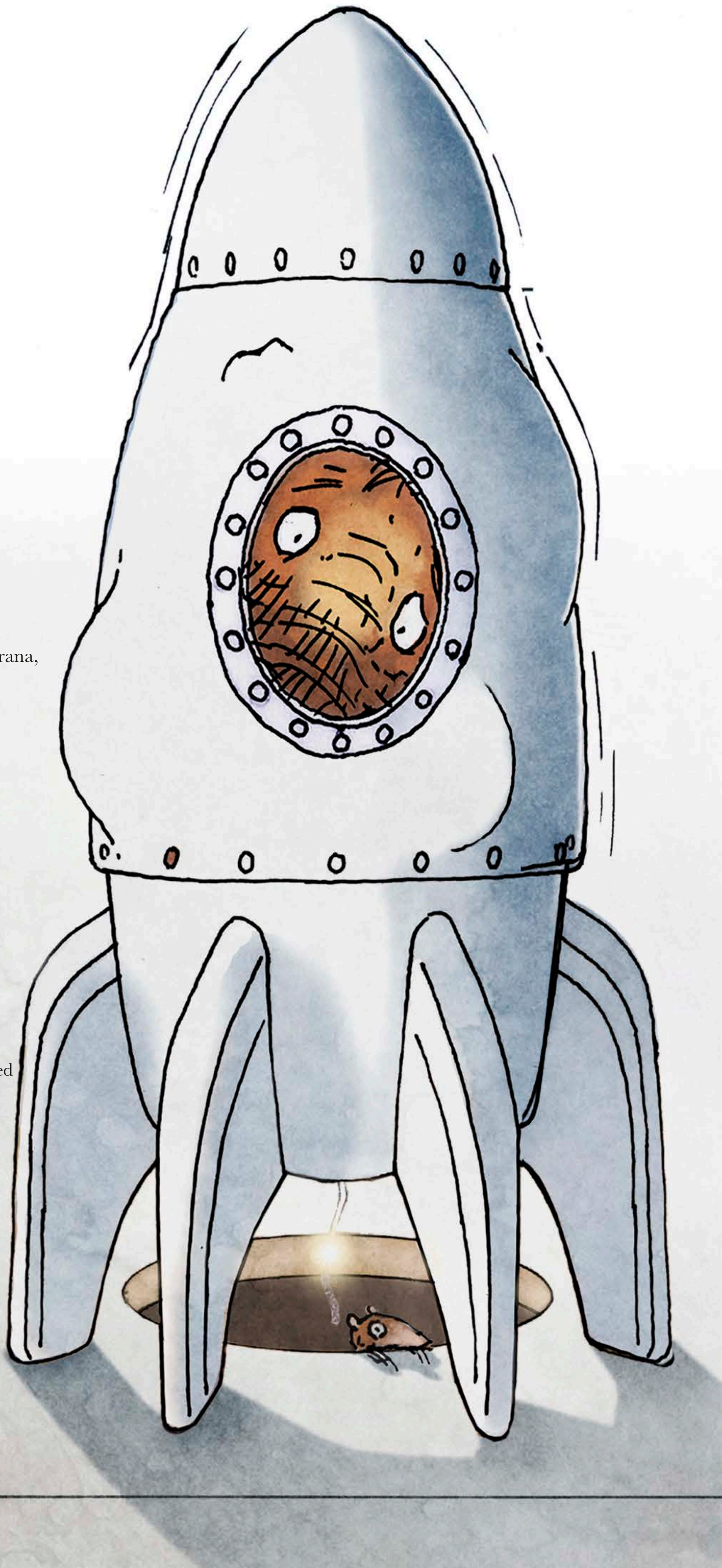

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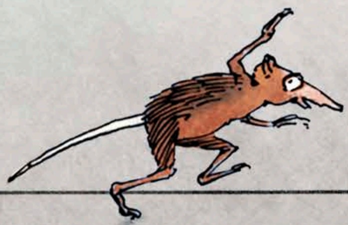
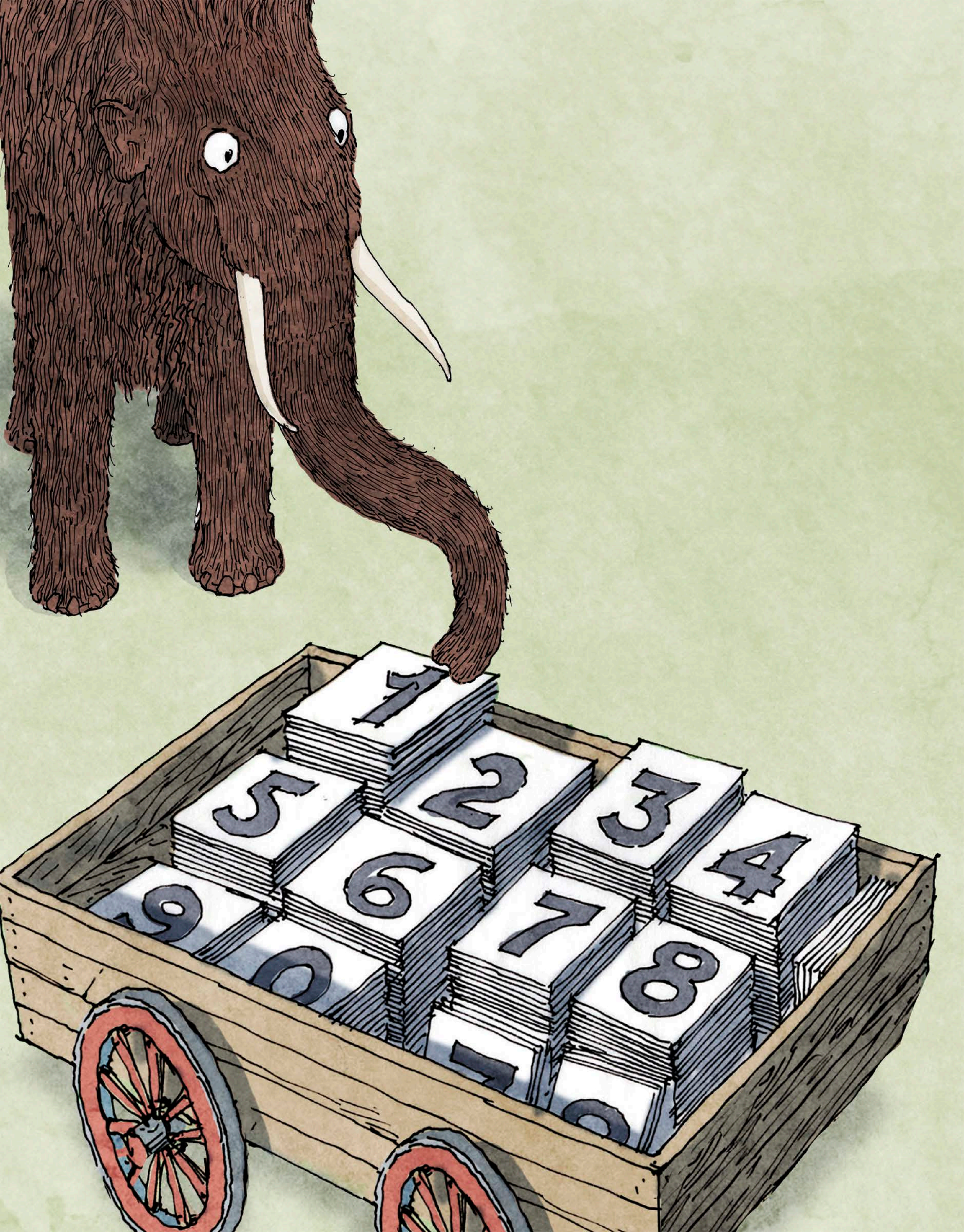



1
2
3
4
5
6
7
8
9
0

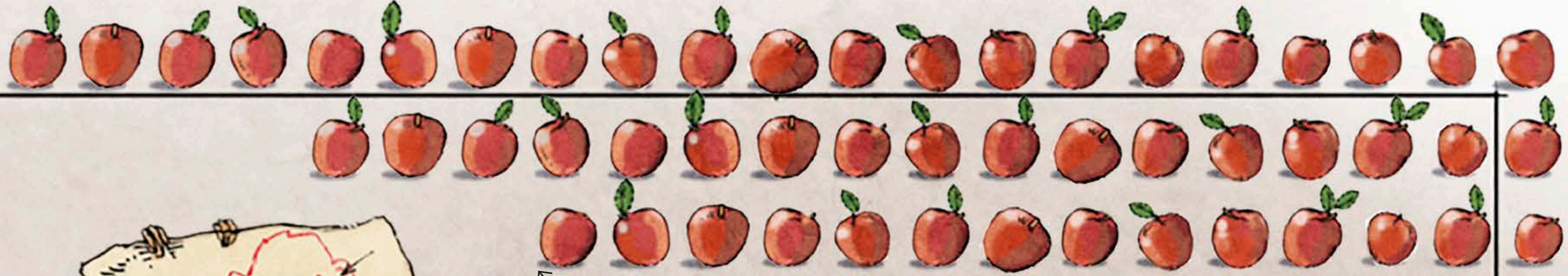
Count on it!

Keeping count

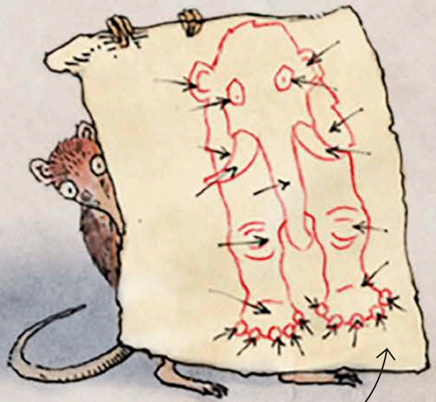
Being able to count is essential for everything from telling the time to keeping score at a football match. Before people had writing or number systems (see pages 12–13), they had to count in their heads (and with whatever they had around to help them). The mammoth and elephant shrews are demonstrating the difficulties of counting their favourite fruit this way, using only bits of their bodies.

Counting in tens
Our number system is based on counting things in groups of 10 – probably because counting on fingers and thumbs is so handy.

Using fingers and toes
Fingers (or mammoth toes) are convenient things to help keep track of numbers less than 10. The shrew works his way from toe to toe, touching one for each apple, counting 8 in total.



Too many apples
With this many objects to keep track of, counting on body parts can be tricky.



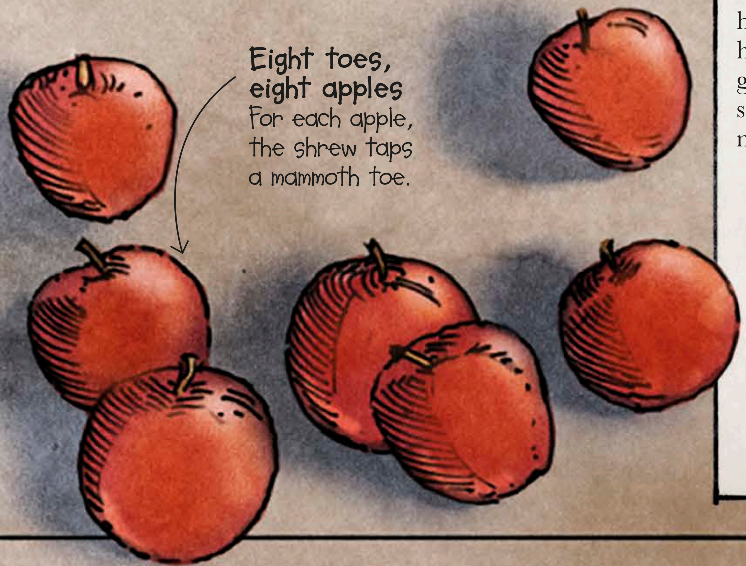
Body work
You could assign numbers to as many body parts as you want – as long as you can remember what they all stand for.

You can count on a mammoth

If you have no words or symbols for numbers, bodies can be useful for helping to count. Touching fingers or other bits of the body as you count helps you keep track. You can also hold fingers up to tell someone else how many of something you have.

Counting beyond 10

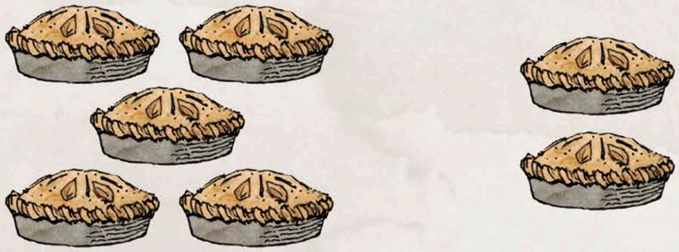
If you need to count numbers larger than 10, you could add more body parts. The mammoth has tried to count a lot of apples like this and got into a muddle. Maybe tallying (see pages 10–11) would work better?



Eight toes, eight apples
For each apple, the shrew taps a mammoth toe.

Counting without counting

Sometimes we know how many things there are without having to count them. With a small group of things, we can often tell how many there are just by looking. This amazing skill we all have is called subitizing. Most of us can easily do this with groups of up to five. And we use it for larger numbers, by seeing smaller groups and adding them together. Can you see how many pies there are here, without counting them?



Tallying to keep count

Counting on fingers, toes, or other body parts is fine, as long you've got a good enough memory to keep track of what you've counted. Keeping a written record is often a much better idea. Tallying is making a stroke or scratch mark for each thing you're counting, such as every time the Sun rises or how many mammoths there are in the herd.

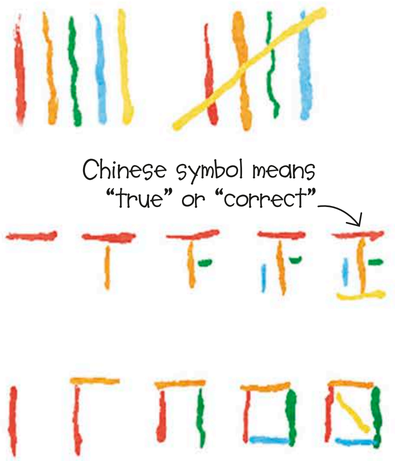
Tally ho!

When you're counting a herd in a hurry, the simplest way is to make one straight line for each mammoth. But all those marks soon add up and become hard to keep track of – imagine how long it would take to count all the marks to get to 100! It's quicker to make groups of marks, and count the groups instead.

Making your mark
Each vertical line represents one mammoth. The elephant shrew draws a line for each mammoth that goes past.

Making tallying easier

Tally marks are still useful today, especially to count things that are moving quickly – like traffic for example. Grouping the marks means you can count groups instead of individual marks, which is quicker and easier. There are different ways of tallying – all these examples show groups of five marks. The first makes a simple "gate" shape. The second builds into a Chinese character. The last method makes a square with a diagonal line through it.




Straight gates
To make it easier to keep track, every fifth tally mark is a diagonal line drawn over the first four marks. These sets of marks are called five-bar gates.
Temporary tallies
If you count a lot of items, you will end up with a lot of marks! To find your total, tot up how many lots of 5 you have. But if your tally gets trampled then it is back to square one.

Number symbols

Numbers were first written down thousands of years ago. Instead of scratching a mark for each item they counted, people assigned each number a symbol instead. Soon, rules were invented so that a small number of symbols could be combined in different ways to make any number imaginable.

Number systems

The elephant shrews are comparing the numbers 1 to 10 in different number systems. Many systems have been invented throughout history, each with its own set of rules for combining symbols to make the number you want to show. The number system we use today was invented in India more than 1,000 years ago.

Ancient Greek
In this system the letters of the alphabet were recycled to stand for numbers, too.

Ancient Roman
This system also used letters, putting them together in different ways to make numbers.

Ancient Chinese
Each number from 1 to 10 had its own symbol, and there were different symbols for multiples of 10: 100, 1,000, and so on.

Hindu-Arabic
The world's most common system is different from those that came before it because it has the symbol zero (0). See pages 16–17 to find out why zero is a hero!

Never-ending numbers

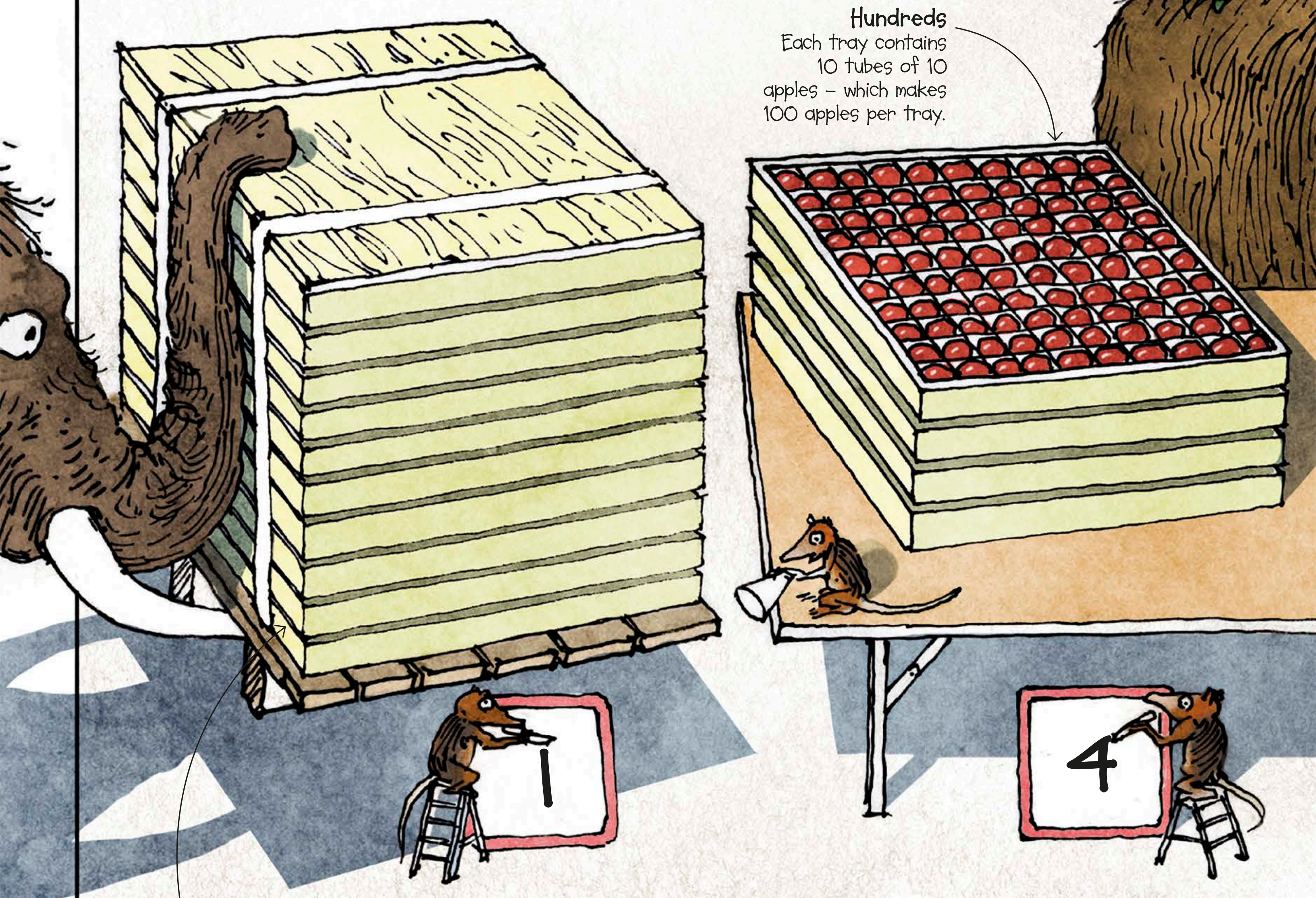
The Hindu-Arabic system uses the digits 1–9 and the symbol 0. Each digit represents a given amount, but these digits can also be used in combination to create an infinite amount of other numbers (see pages 14–15).

Place value

Numbers are made up of symbols called digits: our number system uses the digits 0–9. But the value of these digits can change. For example, in the number 20, the "2" stands for a different amount than it does in the number 200. The amount a digit is worth depends on its position in the number. This is called place value.

Counting in 10s

It's all systems go at the apple-packing plant. Mammoths and elephant shrews are sorting the apples into sets of 10. Each time a set of 10 fills up, it moves a place to the left. Our counting system works like this too – we call it the base-10, or decimal, system. So far, 1,453 apples have been packed.



Thousands
A pallet holds 10 trays, and each tray contains 100 apples. So there are 1,000 apples on this pallet.

One thousand
When there are 10 full trays in the "100s" stack, they move to the left, into the "1000s". There is one full pallet at this station, which means that there are 1,000 apples.

Four hundreds
When the tens trays are full, they move into the hundreds stack. There are 4 full trays here, so that's 4 lots of 100, or 400 apples.

Hold that space!

For the place value system to work, there needs to be a way of showing when a place is empty. This is the special job that zero does (see pages 16–17). In the example below, there are no hundreds in the hundreds column. But without zero to hold the place, we'd be left with 176 – a completely different number.

Thousands	Hundreds	Tens	Ones
1	0	7	6

Five tens
Another mammoth lines up the full tubes in a tray. There are 5 full tubes in the tray so far, which means there are 5 lots of ten, or 50 apples, in the tray.

Three ones
The mammoth puts single apples into a tube. There are 3 apples in the tube, so the elephant shrew writes a "3" on the sign.

Zero

Everyone knows that "zero" means "nothing". But zero isn't just nothing, it's a maths hero with some very important functions. For thousands of years, people did maths without using zero – it was not even considered a number in its own right. Today, it's hard to imagine life without it – things would be very confusing indeed!

Hardworking number

Modern maths could not exist without zero – it is essential to the method of place value that underpins our number system. But everyday life would be much more difficult without zero, too. We need it when we tell the time, take a temperature, or keep score in a sports contest. Here, the mammoths show some of the most useful things that zero does.

Nothing at all

Zero often means "nothing" or "empty", but you can't count to zero – you can't count something that's not there. Look at the pictures above. You wouldn't say there were zero mammoths in the bottom picture, unless you'd already seen the picture above.

Calculating with zero

Zero is the only number on the number line that's neither positive or negative, and neither odd nor even. It is a number that has puzzled mathematicians because it doesn't work quite the same way as other numbers do. For example, you can add, subtract, and multiply with zero, but you can't divide by zero.

$$8 + 0 = 8$$

$$8 - 0 = 8$$

$$8 \times 0 = 0$$

$$8 \div 0 = ????$$

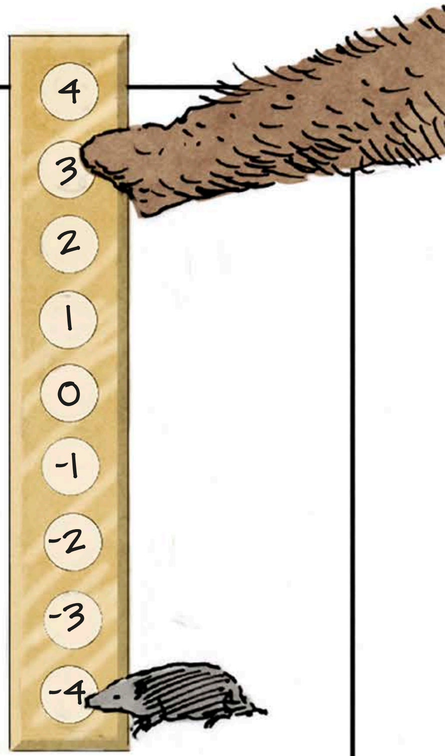
There is no answer that would make sense here

Digital language

Computers communicate in zeros. Binary code is the system we use to give computers their orders: instructions are translated into sequences made up only of 1s and 0s.

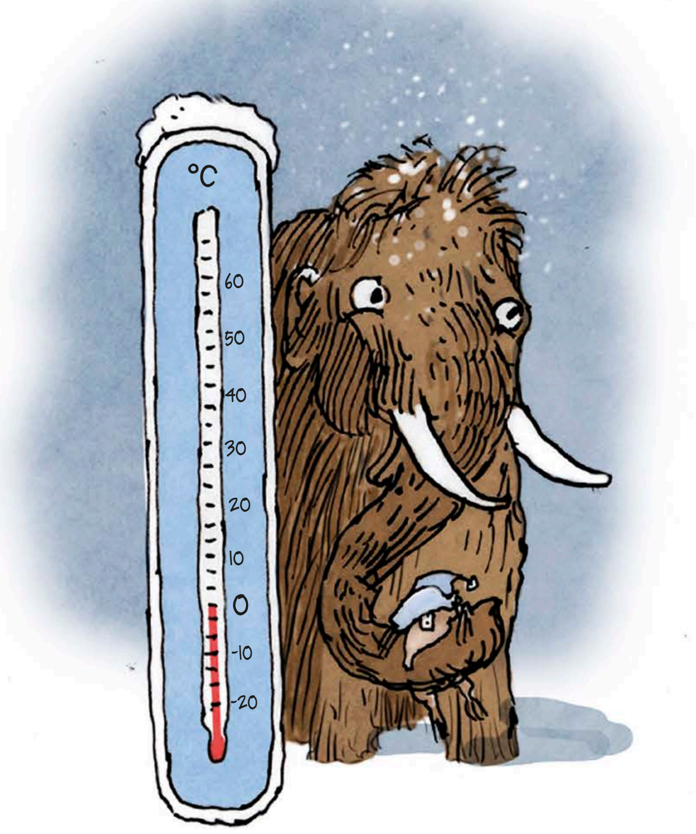
Keeping score

Without zero, it would be harder to keep track in a football match – the "zero" symbol tells us that the blue team haven't scored a goal.



A real number

Zero is a number with its own place on the number line, where it's the dividing point between negative and positive numbers. In a lift, "0" can be used for the ground floor – positive numbers are floors above ground and negative numbers are below ground level.



Taking measurements

When we measure things, zero is a set amount with its own value. The thermometer says 0°, but that doesn't mean there's no temperature – 0° describes a value on the scale.

Showing place value

Zero is essential to our number system. The value of each digit in a number depends on its position (see pages 14–15). Zero "holds the place" of a value when there is no other digit to go in that position.

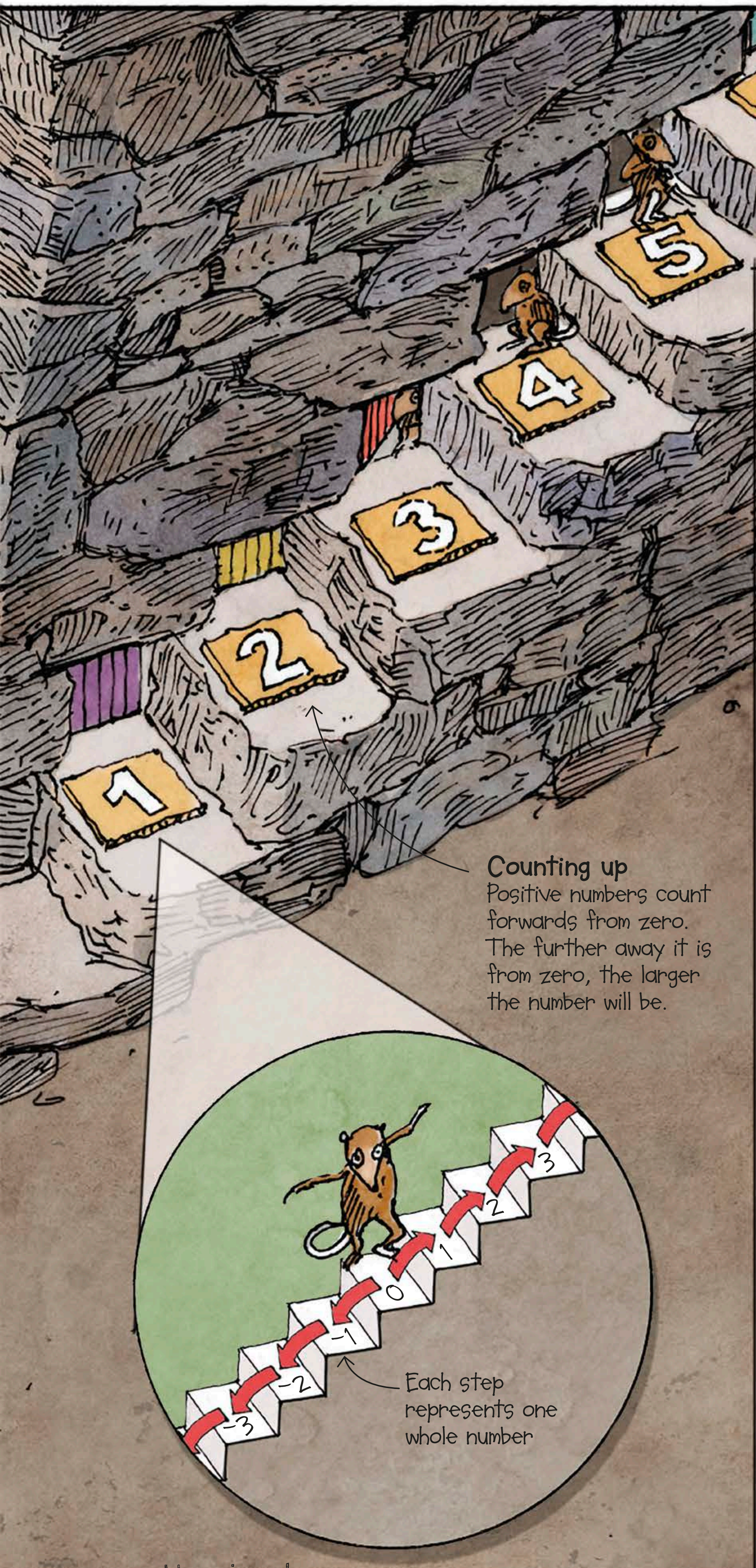
Negative numbers

Any number that is greater than zero is a positive number. If you count down from zero, you go into negative numbers. These are numbers that are less than zero. They are shown with a negative sign (-) in front of them.

A door on every floor

The elephant shrews have built themselves a multi-level housing complex. Each burrow is on a separate floor. Those above ground level (which is marked with a "0") are given positive burrow numbers. The ones beneath ground level have negative numbers on their doormats.

Zero in the middle
Zero (0) is not positive or negative. It's the number that separates positive and negative numbers.



Counting up
Positive numbers count forwards from zero. The further away it is from zero, the larger the number will be.

Each step represents one whole number

Negatives on a number line

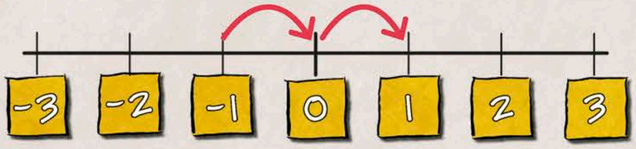
If we simplify the shrews' multi-storey burrows, we can make a number line that helps us to add and subtract positive and negative numbers. Putting the negative number in brackets means it is easier to see its negative symbol. For more on addition and subtraction, see pages 30–33.

Adding a positive number

When you add a positive number to any number, you move to the right on the number line.

$(-1) + 2 = 1$

Move 2 places to the right

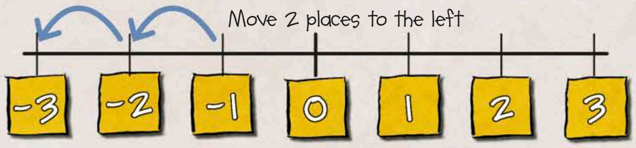


Subtracting a positive number

When you subtract a positive number from another number, you move to the left on the number line.

$(-1) - 2 = (-3)$

Move 2 places to the left

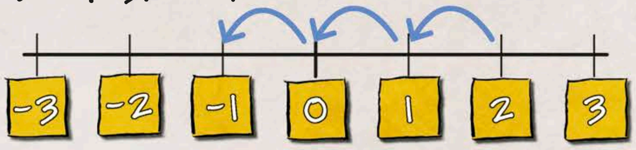


Adding a negative number

When you add a negative number, it's the same as subtracting a positive number: you move to the left.

$2 + (-3) = -1$

Move 3 places to the left

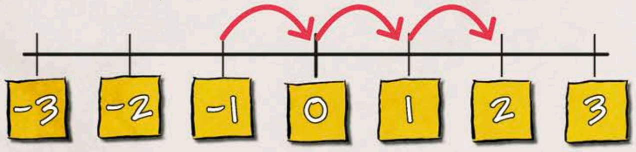


Subtracting a negative number

When you subtract a negative number, it is as if you are adding a positive number: you move to the right.

$(-1) - (-3) = 2$

Move 3 places to the right



Hopping along

The shrew can use the steps like a number line. To count up from zero, the shrew would move to the right, up the steps. To count down from zero, the shrew jumps to the left, down the steps.

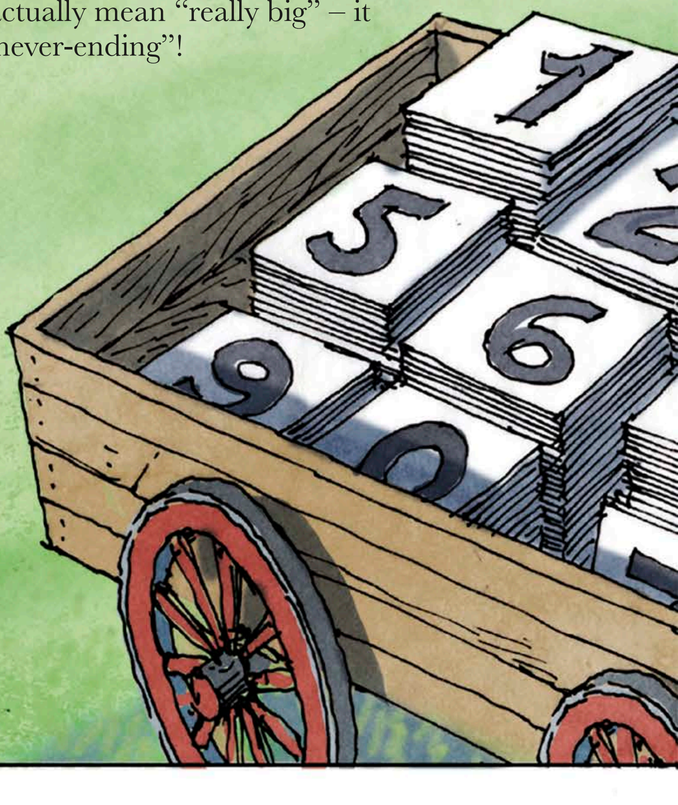
Infinity

What is the biggest possible number? Think of the highest number you know… then add 1 to it. Then try adding another 1. Do it again, and you get another, bigger number. In fact, it's impossible to work out the largest number, because there's no limit to how big (or small) a number can be. In maths, we say that numbers are infinite.

Mind-bogglingly big
The shrews are laying out cards with digits on them, in an attempt to construct a number that goes on forever. The number has become so long, you can't see where it started!

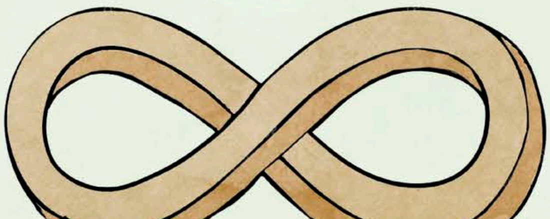
Impossible task

These determined elephant shrews have set out, with the mammoths' help, to create the world's longest number. But no matter how long they stick at it, they'll never succeed, because numbers are infinite. The word "infinite" doesn't actually mean "really big" – it means "never-ending"!



Never-ending symbol

This is the symbol for infinity: it looks like a figure 8 on its side. It's the perfect symbol to use because, like infinity itself, it's got no beginning or end.



Calculations involving infinity don't have the results you might expect. Subtract 1 from infinity and you've still got infinity! This is because infinity isn't actually a number, it's an idea.

$$\infty - 1 = \infty$$

$$50\% \text{ of } \infty = \infty$$

To infinity... and beyond!

It's not just numbers that are infinite – time and space might be too. An infinite amount of time stretching back and forwards forever is called eternity. The Universe contains everything that exists – every galaxy, star, or planet that there ever was or will be. Some scientists think that the Universe is infinite, while others think it has an outer limit and therefore is finite. We'll probably never know for sure.